Frequency-Domain Solution for Coupled Striplines with Crossing Strips

Guang-Wen Pan, Kenneth S. Olson, and Barry K. Gilbert

***Abstract*—In this paper we present a frequency-domain approach to the modeling of the propagation of short-rise-time digital pulses along groups of coupled striplines which are overcrossed or undercrossed by orthogonally positioned signal conductors on adjacent signal planes in a high-density circuit board or multichip module substrate. Although this "crossing strip problem" has been described previously, most recently in a contribution by Gu and Kong [1], the solution presented here has several completely new features which are important in the application of this method to real-world modeling problems in the following ways: First, the new solution significantly simplifies the mathematical formulas which sum the multiple reflections and crosstalk components with the primary digital pulse to generate the final waveform conformations on the multiple conductors (four pages of equations in [1] are reduced to only 16 lines). As a result, this method is much easier to implement than earlier techniques, especially as a software kernel for a computer-aided design tool. The method presented here also reduces the central processing unit (CPU) time needed to execute these solutions by a nontrivial factor of 2–3 in comparison with the earlier method presented by Gu and Kong. Second, the new method removes the earlier constraint that the crossing strips on the orthogonal signal layer be uniformly spaced; that is, nonuniformly spaced crossing strips are now supported by the mathematical derivation. Third, the new derivation allows for nonideal (i.e., "real-world") voltage sources, in contrast to methods described previously (e.g. [1]), which have permitted only ideal step and ramp signals to be directly applied to the signal nets.**

I. Introduction

In this paper we present a frequency-domain approach to the modeling of the propagation of short-rise-time digital pulses along groups of coupled striplines which are overcrossed or undercrossed by orthogonal conductors on adjacent signal planes in a circuit board or multichip module (MCM) substrate.

The problem of modeling the propagation of short-rise-time digital signals along coupled striplines overcrossed or undercrossed by unshielded, orthogonal signal lines has been described several times in the past, most recently in a contribution by Gu and Kong [1]. In that paper, the effects of the crossing lines on the signal-carrying lines were modeled as a change in the characteristic impedance and two small "fringing field" capacitors at the leading and trailing edges of each overcrossing line, as depicted in Figs. 1 and 2. The model used by Gu and Kong was in turn based on a simple but accurate closed-form expression for charge and current distribution on parallel-plate striplines first proposed by Kuester and Chang [2]. The Gu and Kong solution was based on the use of Laplace transforms to determine the transient responses of a coupled pair of striplines having periodic crossing lines not shielded from the striplines. Gu and Kong modeled the effects of crosstalk on the "listening lines," generated by the primary propagating wavefronts on the "driven lines," by the superposition of even- and odd-mode components on the listening lines. A clarification to this paper has also appeared recently [3]. It is also of interest to note that because the lines in this type of structure have frequently been assumed to be lossless and dispersionless, a time-domain solution to this problem is also possible [4], [5]. While these physical models have provided reasonably good approximate results [1], the transient analysis of the propagating waves is rather cumbersome, both in derivation and in software implementation.

Conversely, the solution presented here has several completely new features which are important in the application of this method to real-world modeling problems. One is that the frequency-domain solution proposed here is shown to be much more compact and efficient than either the Laplace transform or the time-domain solutions described by previous workers. That is, this method significantly simplifies the mathematical formulas which sum the multiple reflection and crosstalk components with the primary digital pulse to generate the final waveform conformation on the multiple conductors (four pages of equations in [1] are reduced to only 16 lines). As a result, this method is much easier to implement, especially as a software kernel for a computer-aided design tool. The method presented here also reduces the central processing unit (CPU) time needed to execute these solutions by a not insignificant factor of 2–3 in comparison with the earlier method [1]. A second feature is that the new method removes the constraint that the crossing strips on the orthogonal interconnect layer be uniformly separated from one another. That is, unevenly spaced crossing strips (which are commonly found in printed wiring boards and multichip modules) are now supported by the mathematical derivation. A third feature is that the new derivation allows for nonideal (i.e., "real-world") voltage sources, in contrast to the method described in [1], which permits only ideal step and ramp signals to be directly applied.

II. Formulation

We wish to determine the transient responses at the near and far ends of a pair of coupled transmission lines with k crossing strips. Our method of solution involves two principal steps. First, we establish the odd- and even-mode equivalent circuit models as described in [1]. Second, a frequency-domain solution is presented for the transient responses of the two equivalent circuit models. The resulting waveforms are converted into the time domain via an FFT, and then the voltage responses of the actual structure are found from superposition.

Fig. 2(a) shows the basic configuration of one of the circuit models (for either the even mode or odd mode), represented as a series of transmission line segments cascaded together. The electromotive force, V_{in}, is applied at the far left. We wish to find the voltage response at the near end, V_N, and the far end, V_F, of the equivalent circuit. A discontinuity capacitance, C_d, appears at each line terminal, representing the effect of the edge of a crossing strip. The near-end voltage, V_{0_i}, far-end voltage, V_{D_i}, near-end current, I_{0_i}, and far-end current, I_{D_i}, at the ends of each transmission line segment are described by the following equations (for further explanation, see [6]):

$$V_{0_i} = G_{0_i} + e^{-j\beta_i D_i} G_{D_i} \tag{1}$$

$$I_{0_i} = \frac{1}{Z_i}\left(G_{0_i} - e^{-j\beta_i D_i} G_{D_i}\right) \tag{2}$$

$$V_{D_i} = e^{-j\beta_i D_i} G_{0_i} + G_{D_i} \tag{3}$$

$$I_{D_i} = \frac{1}{Z_i}\left(e^{-j\beta_i D_i} G_{0_i} - G_{D_i}\right) \tag{4}$$

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The authors are with the Mayo Foundation, Rochester, MN 55905.

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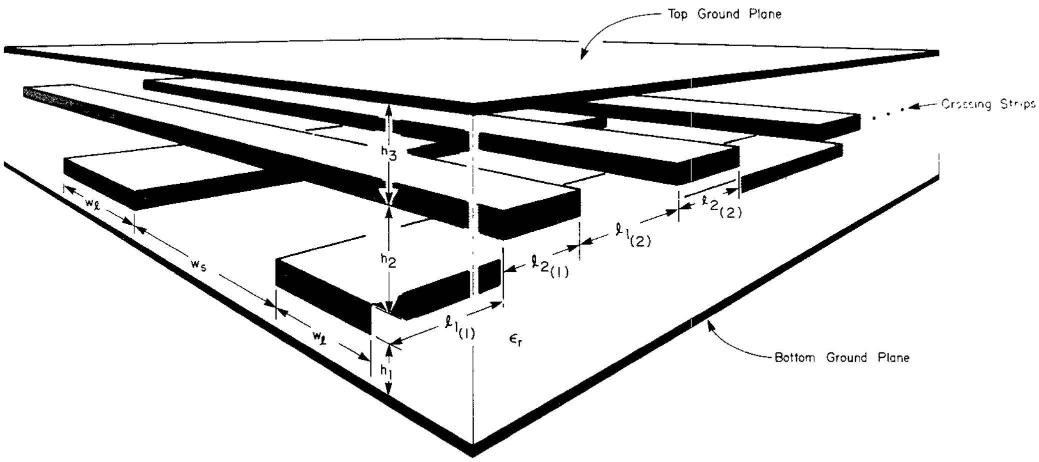

Fig. 1. A pair of coupled striplines with k crossing strips. There is no shielding between orthogonal lines.

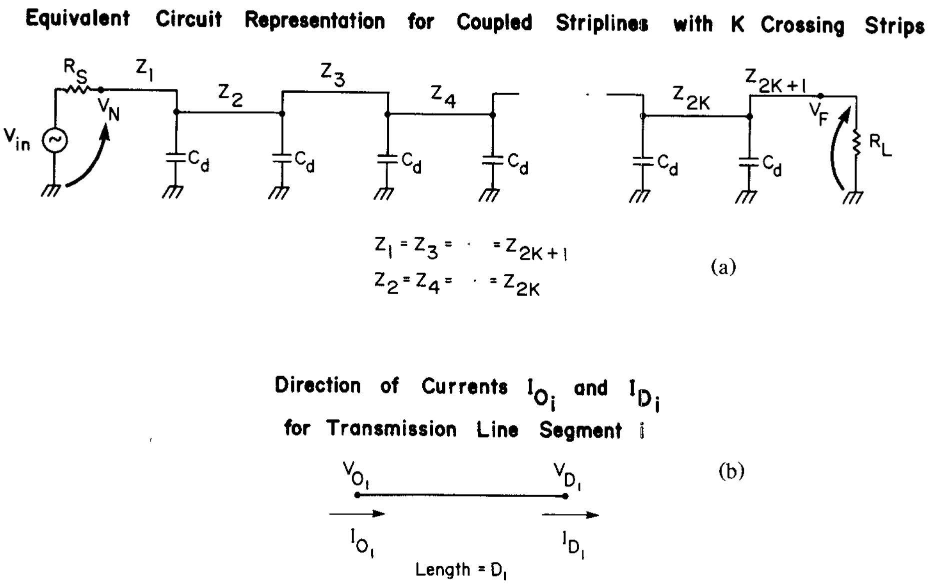

Fig. 2. (a) Even- or odd-mode equivalent circuit. V_N and V_F may be solved for each mode; then, using superposition [7], the actual near-end and far-end voltage responses may be found. (b) Directions of currents I_{0_i} and I_{D_i}. Note that I_{0_i} enters transmission line segment i from the source end and I_{D_i} exits transmission line segment i from the load end. Transmission line segment i has a length D_i.

where i is the transmission line segment under consideration, D_i is the length of line segment i, Z_i is the characteristic impedance of line segment i, $\beta_i = \omega\sqrt{L_i C_i}$, and G_{0_i} and G_{D_i} are defined in [6, eqs. (19)–(22)] as the near-end incident and far-end reflected "modal intensities," respectively, of line segment i. The values of C_i, L_i, Z_i, and C_d may be found from equations presented in [1] and [7]. Fig. 2(b) shows the defined direction of the currents I_{0_i} and I_{D_i} on line segment i. By examining the first transmission line segment, we see that $V_N = V_{0_1}$. Also, by examining the last transmission line segment $(2k+1)$, we note that $V_F = V_{D_{2k+1}}$.

We need to describe mathematically the connectivity of the transmission line segments at each terminal. Once the equations at each line terminal are found (eqs. (5), (7), (8), and (11)), we can incorporate (1)–(4) into them to obtain a set of linear equations with the modal intensities of each line segment (G_{0_i} and G_{D_i}) as the unknowns (eqs. (6), (9), (10), and (12)). Then, once the modal intensities are found, we simply solve (1) and (3) at the line ends to obtain the near-end and far-end modal voltage responses, V_N and V_F (that is, V_{0_1} and $V_{D_{2k+1}}$). The voltage responses of the even and odd modes may be combined as shown in [1] to obtain the actual voltage response waveforms of the active and passive lines.

Looking at the source end of the cascaded transmission line segments in the upper portion of Fig. 2(a), we see that V_{in} can be described (in the frequency domain) in terms of the voltage and current at the beginning of the first transmission line segment (V_{0_1} and I_{0_1}):

$$V_{0_1} + R_S I_{0_1} = V_{\text{in}} \tag{5}$$

where R_S is the source resistance. By substituting the right-hand sides of (1) and (2) into (5), we obtain V_{in} in terms of the modal intensities:

$$V_{\text{in}} = \left(1 + \frac{R_S}{Z_1}\right) G_{0_1} + \left(1 - \frac{R_S}{Z_1}\right) e^{-j\beta_1 D_1} G_{D_1}. \tag{6}$$

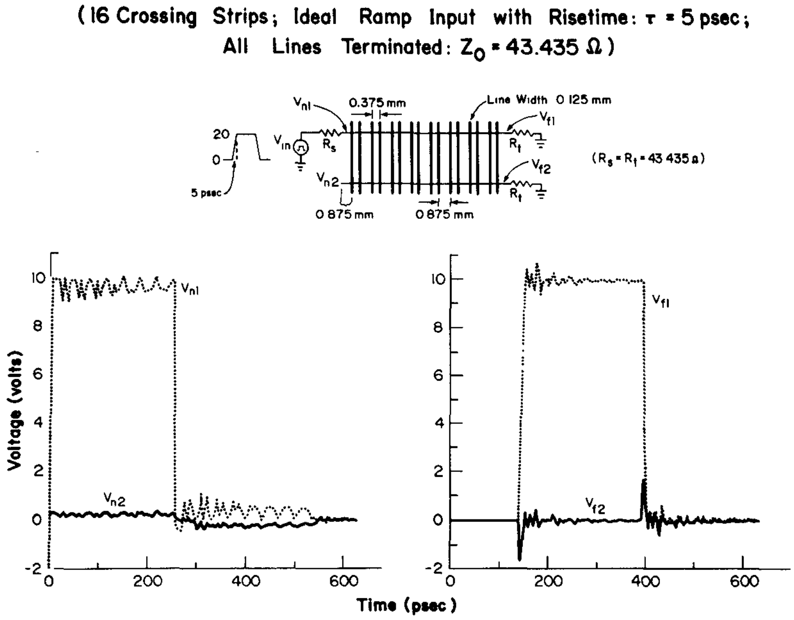

Fig. 3. Transient response for the coupled stripline of Fig. 1 using an alternating (i.e., nonconstant) spacing between the crossing strips of 0.875 mm and 0.375 mm, respectively, with a line width of 0.125 mm. Left panel: Near-end voltages. Right panel: Far-end voltages. The rise time of the input signal is 5 ps.

For each intermediate terminal, two equations can be written, one by equating the voltage at the end of the previous transmission line segment, i, with the voltage at the beginning of the next transmission line segment, $i+1$, and the other by summing the currents leaving the terminal:

$$V_{D_i} = V_{0_{i+1}} \tag{7}$$

$$-I_{D_i} + I_{0_{i+1}} + j\omega C_d V_{D_i} = 0. \tag{8}$$

Equations (7) and (8), rearranged in terms of the modal intensities, become

$$0 = e^{-j\beta_i D_i} G_{0_i} + G_{D_i} - G_{0_{i+1}} - e^{-j\beta_{i+1} D_{i+1}} G_{D_{i+1}} \tag{9}$$

and

$$0 = \left(-\frac{1}{Z_i} + j\omega C_d\right) e^{-j\beta_i D_i} G_{0_i} + \left(\frac{1}{Z_i} + j\omega C_d\right) G_{D_i} + \frac{1}{Z_{i+1}} G_{0_{i+1}} - \frac{1}{Z_{i+1}} e^{-j\beta_{i+1} D_{i+1}} G_{D_{i+1}}. \tag{10}$$

At the load end of the cascaded transmission line system, the voltage and current are related by

$$V_{D_{2k+1}} = V_F = R_L I_{D_{2k+1}} \tag{11}$$

where R_L is the load resistance and k is the number of crossing strips. Equation (11), rewritten in terms of the modal intensities, becomes

$$0 = \left(1 - \frac{R_L}{Z_{2k+1}}\right) e^{-j\beta_{2k+1} D_{2k+1}} G_{0_{2k+1}} + \left(1 + \frac{R_L}{Z_{2k+1}}\right) G_{D_{2k+1}}. \tag{12}$$

If there are k crossing strips in the configuration, then (6), (9), (10), and (12) result in $4k+2$ linear equations with unknowns $G_{0_1}, G_{0_2}, \cdots, G_{0_{2k+1}}$ and $G_{D_1}, G_{D_2}, \cdots, G_{D_{2k+1}}$. This set of equations can be solved for each frequency using a mathematical library such as IMSL. Once the modal intensities are found, the voltages at the near end and the far end, $V_N = V_{0_1}$ and $V_F = V_{D_{2k+1}}$, can be found from (1) and (3). By converting to the time domain (via an inverse FFT), we can then obtain the desired even- and odd-mode near- and far-end voltage responses.

The previous discussions are valid for both the even and the odd mode with the corresponding even and odd parameters. Therefore, the even and odd mode voltages at the near and far ends, namely, $V_N^{(e)}$, $V_N^{(V)}$, $V_F^{(e)}$ and $V_F^{(o)}$, can be evaluated. Finally, the waveforms $V_N^{(1)}$ (at the near end of the active line), $V_F^{(1)}$ (at the far end of the active line), $V_N^{(2)}$ (at the near end of the passive line), and $V_F^{(2)}$ (at the far end of the passive line) can be obtained by the superposition equations [3]:

$$V_N^{(1)} = \frac{1}{2}\left[V_N^{(e)} + V_N^{(o)}\right] \tag{13}$$

$$V_N^{(2)} = \frac{1}{2}\left[V_N^{(e)} - V_N^{(o)}\right] \tag{14}$$

$$V_F^{(1)} = \frac{1}{2}\left[V_F^{(e)} + V_F^{(o)}\right] \tag{15}$$

$$V_F^{(2)} = \frac{1}{2}\left[V_F^{(e)} - V_F^{(o)}\right]. \tag{16}$$

An example demonstrates the ability of the frequency-domain method to simulate more generalized structures such as nonuniform spacing between the crossing strips. In this example, we employed slightly different numerical values for the dimensions in Fig. 1; in particular, we varied the spacing between the crossing strips in an alternation between 0.375 mm and 0.875 mm. The rise time of the signal injected into the driven line was 5 ps. Fig. 3 shows the voltage responses within this nonuniform structure as calculated by the frequency-domain method; 256

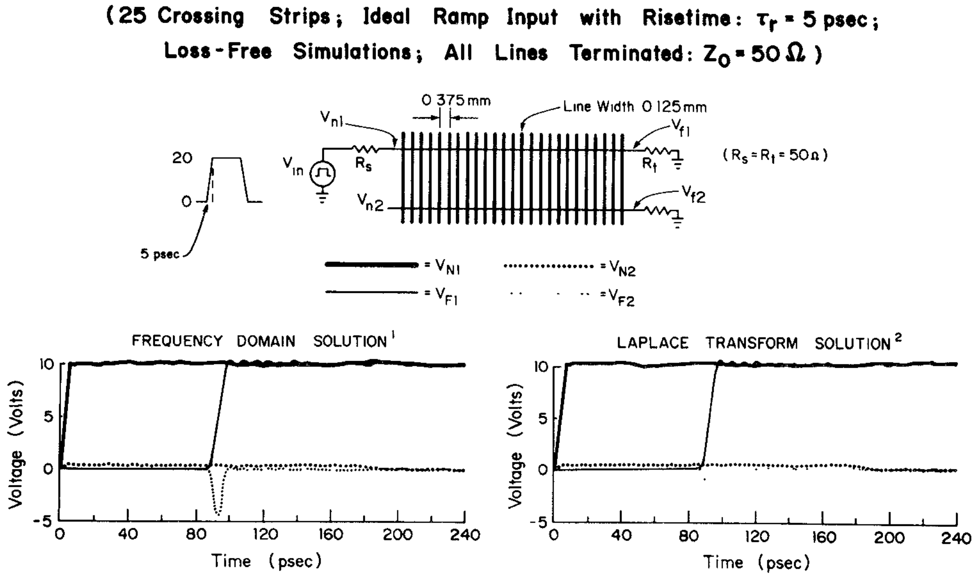

Fig. 4. Simulated waveforms of near-end and far-end voltage transient responses for theoretical coupled stripline structure in a comparison of the frequency-domain method presented here with the Laplace transform solution of [1].

samples were used for the FFT and inverse FFT in this example and in Fig. 4. Of particular interest is the way in which the spacing between crossing strips affects the ringing of the waveform. Looking at $V(N1)$, we see that immediately after the change in logic states, it appears that the duration of each perturbation in the waveform is related to spacing between crossing strips. Continuing in time, the definition of the waveform becomes less clear, as the reflections from additional crossing strips reach the near end of the line.

III. Conclusion

Although the general case of an arbitrary number of asymmetrical lines can be modeled by, for instance, the full-wave solution, the spectral-domain analysis method, or the TLM method, the physical model proposed by Gu and Kong [1] utilizing results from Kuester and Chang [2] still provides a straightforward yet excellent approximation to the problem of parallel striplines with orthogonal, unshielded crossing lines. The present paper has also employed this physical model. A major advantage of this method over [1] and [2] lies in the fact that it significantly reduces the mathematical formulas employed in creating a summation of multiple transmitted and reflected waves, from over four pages of equations [1] down to only 16 lines. Consequently, the new frequency-domain method described here is much easier to comprehend theoretically and, of equal significance and considerable importance, is also much simpler to implement as a software kernel for an electromagnetic modeling computer-aided design (CAD) tool. In addition, any voltage input waveform may be used, as long as it can be transformed accurately into the frequency domain by means of an FFT. In comparison, the Laplace transform solution [1] requires a ramp or step input. Further, as demonstrated by our example, the frequency-domain method can also simulate uneven spacing of the overcrossing strips on the orthogonal signal layer. Thus, the degradation effects of the total number of crossing strips, and of their spacing, can now be observed directly in the simulation results. Uneven line spacing is directly supported because in the determination of the set of $4k+2$ linear equations, each distance D_i may be distinct for each transmission line segment i. Fig. 4 shows a simulation of a coupled stripline structure with 25 evenly spaced crossing strips, in which the frequency-domain method developed here is compared with the Laplace transform solution given in [1]. This example contains evenly spaced crossing strips to show the comparison. The accuracy of our method in comparison with previously published results is evident from this figure.

Time-domain solutions to this type of problem have been described previously; however, in our direct observation, the time-domain solution of this type of electromagnetic structure [4] is not particularly efficient in the solution of the type of structure discussed here. The time-domain methods typically handle only reflections at two ends of the transmission lines (i.e., a "single source, single destination" interconnect), while this method can simulate the effects at $2n+1$ line ends, where n is the number of crossing lines. In addition, while it is true that fully numerical time-domain methods such as the transmission line matrix (TLM) method could be applied to this problem (as noted above), the tremendous amount of computation required makes the TLM method impractical for the most complex (i.e., "typical") structures to be analyzed.

Finally, crosstalk between the parallel lines is treated in our method by means of even–odd mode superposition. To clarify the impact of crosstalk on the groups of lines, we conducted the simulations depicted in Figs. 3 and 4 using the small line widths and interline spacings typical of modern thin-film copper/polyimide multichip module substrates; the crosstalk effects between lines 1 and 2, at both ends of these lines, are clearly visible. This method does, however, ignore crosstalk from each driven line on one signal level to each of the crossing lines on the orthogonal signal layer; this approximation is based on the work of Rubin [8], who made detailed calculations of the magnitude of this crosstalk component and found it to be negligible.

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Spectral-Domain Analysis of Shielded Microstrip Lines on Biaxially Anisotropic Substrates

T. Q. Ho and B. Beker

Abstract —**The spectral-domain technique has been extended to the study of shielded microstrip lines on biaxial substrates. The analysis simultaneously includes both dielectric and magnetic anisotropy effects. A fourth-order formulation leads to the determination of the appropriate Green's function for the structure. The characteristic equation is formed through the application of the Galerkin method to the equations resulting from the boundary conditions on the strip. Numerical results are validated against the data previously published for special isotropic and dielectrically anisotropic cases. New data on the propagation constant of the shielded microstrip with different substrate permittivities and permeabilities are presented to illustrate the effects of the material parameters on the characteristics of the microstrip line.**

I. Introduction

In recent years there has been a steadily growing interest in anisotropic materials for practical uses at millimeter-wave frequencies. The wide variety of possible applications for such media include antenna radomes, substrates for microstrip patch antennas, microwave and millimeter-wave integrated circuits (MIC's), and ferrite nonreciprocal devices. As is well known, the anisotropy in the material may occur naturally or it may be purposely implanted during the fabrication process. In either case, and in particular for MIC's, anisotropic properties of substrates must be included in the analysis, for otherwise serious errors in their design can occur.

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The authors are with the Department of Electrical and Computer Engineering, University of South Carolina, Columbia, SC 29208.
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Since the early works of Owens and Edwards [1], [2], a number of authors have developed different analytical methods for studying transmission lines on anisotropic media. Among these are Alexopolous [3], [4], Horno [5], and Koul *et al.* [6], who used the quasi-static approach to study such problems, while others, among them El-Sherbiny [7], Kobayashi [8], Yang *et al.* [9], and Krowne *et al.* [10], sought full-wave solutions. Although numerous additional works dealing with anisotropic structures are available and are well documented in the literature, the major effort thus far has been directed toward transmission lines with dielectrically anisotropic media. Until now, only a few treatments have been devoted to lines on substrates that are characterized by both $[\epsilon]$ and $[\mu]$ tensors. In one of them, Mariki *et al.* [11] applied the transmission line matrix method to analyze a shielded line on anisotropic substrate. However, no data for magnetic anisotropy effects on propagation constants were provided in that study. On the other hand, for an open structure, Tsalamengas *et al.* [12] used a semianalytical technique which can be used for substrates that are characterized by all nine elements of permittivity and permeability tensors.

In this paper, the spectral-domain method is extended to the study of shielded microstrip lines on biaxially dielectric and magnetic anisotropic substrates. The solution to Maxwell's equations, which for this problem reduces to two coupled second-order differential equations and eventually to two uncoupled fourth-order equations for two components of the electric field, leads directly to the determination of Green's function for the structure. The derivation of the characteristic equation for the propagation constant is carried out using Galerkin's technique in the Fourier domain. To demonstrate numerical efficiency of the spectral-domain approach, results for the convergence studies are included along with samples of the time required for the execution of the code. Numerical results calculated by this method for isotropic as well as dielectrically anisotropic substrates are compared with the existing data, and in both cases a very good agreement is observed. New data for the propagation constant of the shielded lines on substrates simultaneously characterized by different values of $[\epsilon]$ and $[\mu]$ are also generated.

II. Analytical Formulation

Consider the geometry shown in Fig. 1, which illustrates the cross section of the shielded microstrip line situated inside a metal housing along with the coordinate system used in the analysis. Furthermore, the cross section of the structure is assumed to be uniform in the z direction. The metal strip is taken as perfectly conducting and infinitely thin in the x direction. The lossless substrate, which has thickness h_1 and width b, is characterized by homogeneous biaxial permittivity and permeability tensors having the following forms:

$$[\epsilon]=\epsilon_0\begin{vmatrix}\epsilon_{xx} & 0 & 0\\ 0 & \epsilon_{yy} & 0\\ 0 & 0 & \epsilon_{zz}\end{vmatrix} \tag{1a}$$

$$[\mu]=\mu_0\begin{vmatrix}\mu_{xx} & 0 & 0\\ 0 & \mu_{yy} & 0\\ 0 & 0 & \mu_{zz}\end{vmatrix} \tag{1b}$$

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